Second-Order Consensus of Multi-Agent Systems With Delayed Nonlinear Dynamics and Intermittent Measurements

Guanghui Wen and Zhisheng Duan

College of Engineering
Department of Mechanics and Aerospace Engineering
Peking University
Outline

1. Introduction

2. Second-Order Consensus of Multi-Agent Systems With Delayed Nonlinear Dynamics and Intermittent Measurements

3. Applications

4. Conclusions
Consider a multi-agent system consists of \( N \) agents denoted by \( A_i, i = 1, 2, \cdots, N \), each agent \( A_i \) is associated with a coordination variable \( \xi_i(t) \in R^n, i = 1, 2, \cdots, N \). The multi-agent system is said to reach global consensus asymptotically if for any \( \xi_i(0), i = 1, 2, \cdots, N \), \( \|\xi_i(t) - \xi_j(t)\| \to 0 \) as \( t \to \infty \), for all \( i, j \in \{1, 2, \cdots, N\} \).
Typical first-order models

In Olfati-Saber, et al. (2004), the following models are proposed

$$\dot{x}_i(t) = \sum_{j \in N_i} a_{ij}(x_j(t) - x_i(t)), \quad (1)$$

$$x_i(k + 1) = x_i(k) + \epsilon \sum_{j \in N_i} a_{ij}(x_j(k) - x_i(k)). \quad (2)$$

In Ren, et al. (2005), it has been shown that consensus under switching topologies can be achieved if the union of the directed interaction graphs have a spanning tree frequently enough as the system evolves.
Some real dynamical systems require second-order models

- holonomic robots
- relative dynamics of satellites
- harmonic oscillators
- ...
Typical second-order models

In Ren, et al. (2007), the following model is investigated

\[
\begin{cases}
\dot{x}_i(t) = v_i(t), \\
\dot{v}_i(t) = -\sum_{j=1}^{N} l_{ij} x_j(t) - \gamma \sum_{j=1}^{N} l_{ij} v_j(t), & i = 1, 2, \ldots, N.
\end{cases}
\] (3)

In Yu, et al. (2009), some necessary and sufficient conditions are obtained for consensus of

\[
\begin{cases}
\dot{x}_i(t) = v_i(t), \\
\dot{v}_i(t) = -\alpha \sum_{j=1}^{N} l_{ij} x_j(t) - \beta \sum_{j=1}^{N} l_{ij} x_j(t), & i = 1, 2, \ldots, N.
\end{cases}
\] (4)
Second-Order Consensus of Multi-Agent Systems With Intermittent Measurements

- Motivation
  - intermittent communication induced by environmental factors, sensor and actuator failures
  - decreasing communication cost.

- The model

\[
\begin{cases}
  \dot{x}_i(t) = v_i(t), \\
  \dot{v}_i(t) = -\alpha \sum_{j=1}^{N} l_{ij} x_j(t) - \beta \sum_{j=1}^{N} l_{ij} v_j(t), & t \in T, \\
  \dot{v}_i(t) = 0, & t \in \overline{T}, \quad i = 1, 2, \cdots, N,
\end{cases}
\]

where, \( T \cup \overline{T} = [0, +\infty) \).
Main results

**Theorem 1**

Suppose that the communication topology $G(A)$ is strongly connected. Then, second-order consensus in system (5) is achieved if there exists an sequence of time intervals $[t_k, t_{k+1})$, $k \in \mathbb{N}$, with $t_1 = 0$, such that for each time interval $[t_k, t_{k+1})$, $k \in \mathbb{N}$, the following conditions hold:

(i) $a(L) > \alpha/\beta^2$,

(ii) $\delta_k > \frac{\gamma_2}{\gamma_2 + \gamma_3} \omega_k$,

where $\delta_k$ represents the Lebesgue measure of set $\{ t \mid t \in [t_k, t_{k+1}) \cap T \}$, $\omega_k = t_{k+1} - t_k$.

Second-Order Consensus of Multi-Agent Systems With Delayed Nonlinear Dynamics and Intermittent Measurements

Motivation

- each agent may have its intrinsic dynamics: linear or nonlinear, see Arenas, et al. (2008); Ren, (2008); Yu, et al. (2010).
- dynamically switching topology.

Methods

- common Lyapunov function approach
- generalized eigenvalue theory Huang, (1983)
The Model

Our model

\[
\begin{aligned}
\dot{x}_i(t) & = v_i(t), \\
\dot{v}_i(t) & = f(x_i(t-\tau), v_i(t-\tau), x_i(t), v_i(t), t) - \alpha \sum_{j=1}^{m} l_{ij} x_j(t) - \beta \sum_{j=1}^{m} l_{ij} v_j(t), \\
& \quad t \in [k\omega, k\omega+\delta], \\
\dot{v}_i(t) & = f(x_i(t-\tau), v_i(t-\tau), x_i(t), v_i(t), t), \quad t \in (k\omega+\delta, (k+1)\omega), \\
& \quad k \in \mathbb{N}, \quad i = 1, 2, \ldots, N.
\end{aligned}
\]
Assumption 1

There exist nonnegative constants $\rho_i, i \in \{1, 2, 3, 4\}$, such that

$$
\| f(x_1, x_2, x_3, x_4, t) - f(y_1, y_2, y_3, y_4, t) \| \leq \sum_{i=1}^{4} \rho_i \| x_i - y_i \|,
$$

$\forall x_i, y_i \in \mathbb{R}^n, i \in \{1, 2, 3, 4\}, t \geq 0.$
Main Results—Fixed Topology Case

**Theorem 2**

Suppose that the communication topology $\mathcal{G}(\mathcal{A})$ is strongly connected and balanced, and Assumption 1 holds. Then, second-order consensus in system (6) is achieved if $\beta > \alpha$, and the following conditions hold:

1. $\lambda_2(L + L^T) > \max \{ \alpha^{-1}, \varrho_1, \varrho_2 \}$,
2. $\delta > \frac{r \tau + (\gamma_2 + \gamma_3) \omega}{r + \gamma_2 + \gamma_3}$,

where

\[ \varrho_1 = \frac{\alpha(\rho_1 + \rho_2 + 2\rho_3) + \beta \rho_3 + \alpha \rho_4}{\alpha^2} + \frac{\max\{\rho_1, \rho_2\}(\alpha + \beta)(\beta \lambda_N (L + L^T) + 1)}{\alpha(\beta - \alpha)}, \]

\[ \varrho_2 = \frac{\beta(\rho_1 + \rho_2 + 2\rho_4) + 2\alpha}{\beta^2} + \frac{\max\{\rho_1, \rho_2\}(\alpha + \beta)[\alpha \beta \lambda_N (L + L^T) + \alpha]}{\beta^2(\beta - \alpha)}. \]
Main Results—Switching Topology Case

**Theorem 3**

Suppose that the communication topology $G(A_{\sigma(t)})$ is kept strongly connected and balanced throughout the process and, moreover, Assumption 1 holds. Then, second-order consensus in system (5) is achieved if $\beta > \alpha$ and the following conditions hold:

(i) $\min_{i \in \Pi} \lambda_2(L_i + L_i^T) > \max \{ \alpha^{-1}, \varrho_0 \}$,

(ii) $\delta > \frac{\bar{r}\tau + (\bar{\gamma}_2 + \bar{\gamma}_3)\omega}{\bar{r} + \bar{\gamma}_2 + \bar{\gamma}_3}$,

where $\varrho_0 = \max_{i \in \Pi} \left\{ \varrho_1^i, \varrho_2^i \right\}$,

\[
\varrho_1^i = \frac{\alpha(\rho_1 + \rho_2 + 2\rho_3) + \beta \rho_3 + \alpha \rho_4}{\alpha^2} + \frac{\rho_0(\alpha + \beta)(\beta \lambda_N (L_i + L_i^T) + 1)}{\alpha(\beta - \alpha)},
\]

\[
\varrho_2^i = \frac{\beta(\rho_1 + \rho_2 + 2\rho_4) + 2\alpha}{\beta^2} + \frac{\rho_0(\alpha + \beta)[\alpha \beta \lambda_N (L_i + L_i^T) + \alpha]}{\beta^2(\beta - \alpha)}.
\]
The time-delayed nonlinear function $f$ is described by time-delayed Chua’s circuit Wang, et al. (2000):

$$f(x_i(t - \tau), v_i(t - \tau), x_i(t), v_i(t), t) = \begin{pmatrix} \mu(-v_{i1} + v_{i2} - l(v_{i1})) \\ v_{i1} - v_{i2} + v_{i3} \\ -\zeta v_{i2} - \epsilon \sin(\sigma v_{i1}(t - \tau)) \end{pmatrix}, \quad (7)$$

where $l(v_{i1}) = b v_{i1} + 0.5(a - b)(|v_{i1} + 1| - |v_{i1} - 1|)$, $x_i = [x_{i1}, x_{i2}, x_{i3}]^T$, $v_i = [v_{i1}, v_{i2}, v_{i3}]^T$. The isolated system (7) is chaotic when $\mu = 10$, $\zeta = 18$, $\epsilon = 0.02$, $\sigma = 0.02$, $\tau = 0.01$, $a = -4/3$ and $b = -3/4$. In view of Assumption 1, one obtains $\rho_1 = 0$, $\rho_2 = 0.0004$, $\rho_3 = 0$, $\rho_4 = 4.3871$. Let $\alpha = 11.5$, $\beta = 12$, $\delta = 0.48$, and $\omega = 0.5$. By simple calculations, one has the consensus conditions given in Theorem 3 are satisfied.
Figure: Communication topology $G(A_i), \ i = 1, 2$. 
Figure: Consensus of state trajectories of multiple agents.
Figure: Consensus of velocity trajectories of multiple agents.
The relative dynamics of the $i$th satellite with respect to the virtual satellite are given by Hill’s equations, which are

$$\ddot{x}_i - 2\omega_0 \dot{y}_i = u\tilde{x}_i$$
$$\ddot{y}_i + 2\omega_0 \dot{x}_i - 3\omega_0^2 \tilde{y}_i = u\tilde{y}_i$$
$$\ddot{z}_i + \omega_0^2 \tilde{z}_i = u\tilde{z}_i$$

where $\tilde{x}_i$, $\tilde{y}_i$, and $\tilde{z}_i$ are the position components of the $i$th satellite in the rotating coordinate; $u\tilde{x}_i$, $u\tilde{y}_i$, and $u\tilde{z}_i$ are the control inputs; and $\omega_0$ denotes the angular rate of the virtual satellite.
Denote the position vector by $r_i = [\tilde{x}_i, \tilde{y}_i, \tilde{z}_i]^T$ and the control vector by $v_i = [\dot{\tilde{x}}_i, \dot{\tilde{y}}_i, \dot{\tilde{z}}_i]^T$. Then, (8) can be rewritten as

$$\begin{cases}
\dot{r}_i = v_i, \\
\dot{v}_i = Ar_i + Bv_i + \alpha \sum_{j=1}^{m} a_{ij} (r_j - h_j - r_i + h_i) + \beta \sum_{j=1}^{m} a_{ij} (v_j - v_i),
\end{cases}$$

$$t \in [k\omega, k\omega + \delta],$$

$$\dot{v}_i = Ar_i + Bv_i, \quad t \in (k\omega + \delta, (k+1)\omega), \quad k \in \mathbb{N}, \quad i = 1, 2, \cdots, N,$$

where $A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 3\omega_0^2 & 0 \\ 0 & 0 & -\omega_0^2 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 2\omega_0 & 0 \\ -2\omega_0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. 
Let $\omega_0 = 0.001$, $r_1(0) = (1, 0.5, 2)^T$, $v_1(0) = (0, -1, 0)^T$, $r_2(0) = (2, 2, -1)^T$, $v_2(0) = (0.2, 0, 1)^T$, $r_3(0) = (-1, -3, 5)^T$, $v_3(0) = (0.5, 0, -0.2)^T$, $r_4(0) = (3, 4, 2)^T$, $v_4(0) = (0, -0.5, 1)^T$, $h_1 = (2, 2, 0)$, $h_2 = (-2, 2, 0)$, $h_3 = (-2, -2, 0)$, $h_4 = (2, -2, 0)$. By Theorem 3, consensus can be achieved if $\delta/\omega > 15.50\%$. In simulations, let $\omega = 1.0s$, $\delta = 0.25s$, $\alpha = 11.5$, and $\beta = 12$.

**Figure:** Communication topology $\mathcal{G}(\mathcal{A})$. 
Figure: Relative positions of the four satellites.
Figure: Relative velocities of the four satellites.
Conclusions

- stochastic intermittent communication
- higher-order consensus for multi-agent systems with nonlinear dynamics
- attitude synchronization for multiple rigid bodies
Collaborators

- Guanrong Chen, City University of Hong Kong
- Wenwu Yu, Southeast University


Other Collaborators

- Xianmin Geng, Nanjing University of Aeronautics and Astronautics
- Zhongkui Li, Beijing Institute of Technology

Some pictures are collected from the Internet.
References


Thanks!